INTEGRATED MSc-PhD PROGRAM

KERALA SCHOOL OF MATHEMATICS
Kunnamangalam
Kozhikode - 673571

Contents

1. General Information

1.1. Kerala School of Mathematics. Kerala school of Mathematics (KSoM) – a joint initiative of the Kerala State Council for Science, Technology and Environment, Government of Kerala and the Department of Atomic Energy, Government of India – is a national institute for advanced research in Mathematics. KSoM was founded with the vision of establishing a centre of excellence for research in Mathematics. The institute is a meeting ground for leading mathematicians from across the world and a centre of advanced learning in Mathematics in the state of Kerala.

1.2. The programme. The Integrated MSc-PhD programme in Mathematics at KSoM is an advanced training program at the postgraduate level leading to doctoral studies. The coursework is very flexible with a focus on grooming students for a research career in Mathematics. Depending on the choice of the electives, a student also has the option to choose a research career in Physics or Theoretical Computer Science as well. The curriculum is among the best that is available in the country at the postgraduate level. The *Board of Studies for the Integrated MSc-PhD programme of KSoM* will supervise the Integrated MSc-PhD programme.

1.3. Admission. The admission for the Integrated MSc-PhD programme at KSoM will be based on the national level entrance examination conducted by CMI. Shortlisted candidates will be called for a test/interview thereafter at KSoM. The minimum qualification required for joining the Integrated MSc-PhD programme is B.Sc., B.Math., B.Stat. or an equivalent degree with Mathematics as one of the subjects or BE / B.Tech. or equivalent degree.

1.4. Duration. The duration of the Integrated MSc-PhD programme shall be a minimum of ten semesters and a maximum of sixteen semesters. The first four semesters are dedicated to the master's level coursework and the remaining towards doctoral studies leading to a PhD. An academic year has two semesters, one from July/August to November and the other from January to May. There will be a break between two semesters

1.5. Scholarship. The students selected for the Integrated MSc-PhD programme are entitled for a scholarship. The scholarship amount is currently Rs. 5000 during the MSc coursework for a maximum period of four semesters. However, a student who does not secure at least 26 credits in the first year forfeits her/his scholarship. The scholarship during PhD will be at par with NBHM PhD scholarship.

2. Course structure

2.1. A total of 76 credits in two years is required to obtain the MSc. degree. Every student has to take 8 core courses compulsorily and obtain 32 credits. The number of credits allocated to any course is equal to the number of contact hours every week including tutorial sessions in that particular course. In particular, every core course will have at least four contact hours every week.

2.2. The elective courses offered at KSoM are listed in subsection 4.2. All the courses listed there may, however, not be offered in an academic year. The list of courses offered every year will be announced at the beginning of every academic year and will be listed in the institute website. The students may also credit any course offered in any *Institute of Excellence* as defined in the regulations of the University of Calicut for this programme. This can be done only after approval of the same by the Board of Examinations(described in 2.6).

2.3. Every student should decide on the elective courses to be credited in a particular semester in consultation with the Board of Examinations. The same must be done one week prior to the beginning of every semester.

2.4. Every student has to work on a Master's project in the fourth semester to complete the MSc. degree requirements. It involves the study of an advanced topic or a research paper. A student may also pursue her/his Master's project under the supervision of a faculty member from an *Institute of Excellence* as described in the regulations of the University of Calicut for this programme. The same must be done only with the approval of the Board of Examinations.

2.5. A Viva Voce is conducted at the end of the first three semesters. This will be a comprehensive oral examination based on the subjects credited by a particular student in that semester.

2.6. Board of Examinations. The Board of Examinations will oversee the functioning of the MSc. programme. The committee will consist of three faculty members of KSoM.

2.7. A student may continue into the PhD programme subject to the the following conditions :

(1) The student has acquired 76 credits at the end the fourth semester.

(2) The *CGPA* of the student is at least 8(in case of SC/ST candidates the cut-off shall be 7).

(3) The student has not obtained a grade of **B+** or **C** in more than four courses during the MSc coursework.

2.8. The course structure for the MSc is listed below. The detailed syllabi for the courses offered at KSoM are listed in subsections 4.1 and 4.2. The number of credits are indicated in brackets.

Semester 1	**Semester 2**
Algebra-1 (4)	Algebra-2 (4)
Analysis-1 (4)	Analysis-2 (4)
Topology (4)	Complex Analysis (4)
Probability Theory (4)	Differential Equations (4)
Viva voce (4)	Viva Voce (4)

Semester 3	**Semester 4**
Elective-1 (4)	Elective-5 (4)
Elective-2 (4)	Elective-6 (4)
Elective-3 (4)	Master's project (8)
Elective-4 (4)	
Viva voce (4)	

3. Evaluation and promotion

3.1. Evaluation scheme for courses offered at KSoM will contain two components : Continuous Assessment and End-semester Examination. The weightage of continuous assessment and end-semester examination in any course offered at KSoM will be as given below.

	Core Course	Elective course
Continuous Assessment	50%	40%-60%
End-Semester Examination	50%	40%-60%

The weightage allotted to continuous assessment and end semester examination in an elective course will be declared at the beginning of the course by the instructor(s).

3.2. Evaluation will be based on seven letter grades (**O**, **A+**, **A**, **B+**, **B**, **C** and **F**) with numerical values (grade points) of 10, 9, 8, 7, 6, 5 and 0 respectively. In addition to the above, an incomplete grade given by **I** and an absent grade given by **Ab** will also be used. The grading scheme involved in any course at KSoM is absolute.

3.3. The continuous assessment for the courses offered at KSoM will be based on a predetermined transparent system involving the following components :

	Core Course	Elective course
Mid-semester examinations	20%-30%	20%-30%
Periodic written tests	0%-20	0%-20%
Assignments	10%-30%	10%-30%
Seminars or other components		0%-20%

The transparent system will be declared at the beginning of the course by the instructor(s). The end-semester examinations for every course will be conducted and evaluated by the instructor(s) of the course.

3.4. Promotion policy. The grades are divided in to the following categories.

Grade	*Grade Points*	Marks range	Merit
O	10	90-100	Outstanding
A+	9	80-89.99	Very good
A	8	70-79.99	Good
B+	7	60-69.99	Above average
B	6	55-59.99	Average
C	5	50-54.99	Pass
F	0	Below 50	Fail
I	0		Incomplete
Ab	0		Absent

3.5. In the case when a course is offered in an *Institute of Excellence*, the evaluation scheme that will be followed will be as per the norms followed in the relevant *Institute of Excellence*. The respective instructor(s) from the *Institute of Excellence* shall be requested to transfer the score and grade obtained by each student to the *Board of Examinations*. The *Board of Examinations* will adapt and normalize the score and grade obtained by each student in such courses as per the table above.

3.6. A student securing the grade ‘F’ in the respective course will be treated as having failed in the course. In such a scenario an option of a make-up examination will be provided. The make-up examination will be held in the week before the beginning of the next semester. The Cumulative Grade Point Average (CGPA) out of 10 is calculated as per the following formula

$$\text{CGPA} = \frac{\sum_{\text{Courses taken in the programme}} (\text{No. of credits of the course} \times \text{Grade Points})}{\text{Net total number of credits obtained in the programme}}.$$

A semester grade point average (SGPA) is calculated similarly every semester.

3.7. A student who has obtained 76 credits at the end of the second year, irrespective of satisfying the eligibility criterion for promotion into PhD, will be given an MSc degree.

3.8. The maximum time for completion of the MSc is 4 years during which 76 credits should be acquired. A student needs to acquire a minimum of 20 credits to get promoted to the second year of the programme. Any student unable to do so in the first two years will be asked to discontinue the MSc. programme.

4. Syllabus

4.1. Core Courses. Every student has to complete 32 credits from core courses compulsorily. The syllabi for the core courses are detailed below.

4.1.1. *KSM1C01 : Algebra I.*

Unit I

Matrix operations: row reductions, determinants, Cramer's rule.

Unit II

Vector spaces: real and complex vector spaces, bases and dimension, computation with bases, direct sums, Linear transformations: dimension formula.

Unit III

Matrix of a linear transformation, linear operators and eigenvalues, characteristic polynomial, Jordan canonical form, Inner product spaces and bilinear forms, Gram-Schmidt orthonormalization, Hermitian forms, diagonalization.

Unit IV

Self-Adjoint and Normal Operators, Adjoints, Self-Adjoint Operators, Normal Operators, Spectral Theorem, Positive Operators, Isometries.

Suggested texts :

(1) Michael Artin, *Algebra*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1991.
(2) Kenneth Hoffman and Ray Kunze, *Linear algebra*, Prentice-Hall Mathematics Series, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1961.
(3) Denis Serre, *Matrices*, second ed., Graduate Texts in Mathematics, vol. 216, Springer, New York, 2010, Theory and applications.
(4) Sheldon Axler, *Linear algebra done right*, third ed., Undergraduate Texts in Mathematics, Springer

4.1.2. *KSM1C02 : Analysis I.*

Unit I

Metric Spaces, Convergent Sequences, Cauchy Sequences, Series, The Root and Ratio Tests, Power Series, Limits of Functions Continuous Functions

Unit II

The Derivative of a Real Function, Mean Value Theorems, The Continuity of Derivatives, Taylor's Theorem, Definition and Existence of the Integral, Properties of the Integral, Integration and Differentiation.

Unit III

Functions of Several Variables, Linear Transformations, Differentiation, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem, The Rank Theorem.

Unit IV
Determinants, Derivatives of Higher Order, Differentiation of Integrals, Integration.

Suggested texts :

(1) Walter Rudin, P*rinciples of mathematical analysis*, third ed., McGraw-Hill Book Co., New York-Auckland-DÃijsseldorf, 1976, International Series in Pure and Applied Mathematics.
(2) Terence Tao, *Analysis. I*, third ed., Texts and Readings in Mathematics, vol. 37, Hindustan Book Agency, New Delhi
(3) Tom M. Apostol, *Mathematical analysis*, second ed., Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1974.
(4) James R. Munkres, *Analysis on manifolds*, Addison-Wesley Publishing Company, Advanced Book Program, Redwood City, CA, 1991.

4.1.3. *KSM1C03 : Topology.*

Unit I
Topological Spaces and Continuous Functions, Connectedness and compactness, Tychonoff's theorem,
Unit II
Countability and Separation Axioms, normal and regular spaces, Urysohn-Tietze theorems.
Unit III
Fundamental groups, Van Kampen's theorem,
Unit IV
Covering space theory.

Suggested texts :

(1) James R. Munkres, *Topology*, Prentice Hall, Inc., Upper Saddle River, NJ, 2000, Second edition
(2) I. M. Singer and J. A. Thorpe, *Lecture notes on elementary topology and geometry*, Springer-Verlag, New York-Heidelberg, 1976, Reprint of the 1967 edition, Undergraduate Texts in Mathematics.

(3) Allen Hatcher, *Algebraic topology*, Cambridge University Press, Cambridge, 2002.
(4) George F. Simmons, *Introduction to topology and modern analysis*, McGraw-Hill Book Co., Inc., New York-San Francisco, Calif.-Toronto- London, 1963.

4.1.4. *KSM1C04 : Probability Theory.*

Unit I
Probability space (Countable sample spaces), Inclusion-exclusion principle, Occupancy and Matching problems with Poisson asymptotics, Conditional Probability and Independence, Polya's urn scheme - exchangeability.
Unit II
Random variables and their distributions, Bernoulli, Binomial, Geometric, Poisson, Hypergeometric distributions, Expectation of a random variable. Continuous distributions – An introduction, Uniform, Normal, Exponential, Gamma, Beta distributions, Simulating random variables on a computer.
Unit III
Variance and Moments, Joint distribution and independence of random variables, Conditional distributions, Markov's and Chebyshev's inequalities, Weak law under second moment assumption, CLT for sums of Bernoullis, Poisson limit for rare events.
Unit IV
Simple random walk in one dimension, Ballot problem, Gambler's ruin, Recurrence,

Suggested texts :

(1) William Feller, *An introduction to probability theory and its applications. Vol. I*, Third edition, John Wiley & Sons, Inc., New York-London-Sydney, 1968.
(2) William Feller, *An introduction to probability theory and its applications. Vol. II*, Second edition, John Wiley & Sons, Inc., New York-London-Sydney, 1971.
(3) Sheldon Ross, *A first course in probability*, second ed., Macmillan Co., New York; Collier Macmillan Ltd., London, 1984.

4.1.5. *KSM2C01 : Algebra II.*

Unit I
Group theory: group actions, Sylow theorems.
Unit II
Ring theory: commutative rings and homomorphisms, polynomial rings.
Unit III
Euclidean rings, PIDs and UFDs.

Unit IV
Modules, semisimplicity, Wedderburn theorem

Suggested texts :

(1) Michael Artin, *Algebra*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1991.
(2) Serge Lang, *Algebra*, third ed., Graduate Texts in Mathematics, vol. 211, Springer-Verlag, New York, 2002.
(3) David S. Dummit and Richard M. Foote, *Abstract algebra*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1991.
(4) Thomas W. Hungerford, *Algebra*, Graduate Texts in Mathematics, vol. 73, Springer-Verlag, New York-Berlin, 1980, Reprint of the 1974 original.

4.1.6. *KSM2C02 : Analysis II.*

Unit I
Measure spaces, monotone classes, outer measures, Lebesgue measure, Regularity, measurable functions, integration, Monotone Convergence Theorem, Dominated Convergence Theorem.
Unit II
Product measures, Fubini, Radon-Nikodym derivative.
Unit III
Normed linear spaces, boundedness, completeness of $B(X,Y)$, $C(X)$, Hahn-Banach (statement only proof later), Heine-Borel, all norms are equivalent in finite dimensions, Hilbert spaces, Cauchy-Schwartz inequality, existence of orthonormal basis, Riesz lemma.
Unit IV
Basic Fourier analysis on the circle and on R (up to Plancheral theorem). If time permits: Cantor set, complex measures, Riesz representation theorem.

Suggested texts :

(1) Walter Rudin, *Real and complex analysis*, third ed., McGraw-Hill Book Co., New York, 1987.
(2) Gerald B. Folland, *Real analysis*, second ed., Pure and Applied Mathematics (New York), John Wiley & Sons, Inc., New York, 1999, Modern techniques and their applications, A Wiley-Interscience Publication.

(3) Elias M. Stein and Rami Shakarchi, *Real analysis*, Princeton Lectures in Analysis, vol. 3, Princeton University Press, Princeton, NJ, 2005, Measure theory, integration, and Hilbert spaces.
(4) Alberto Torchinsky, *Real variables*, Addison-Wesley Publishing Company, Advanced Book Program, Redwood City, CA, 1988.

4.1.7. *KSM2C03 : Complex Analysis.*

Unit I
Complex numbers and geometric representation, analytic functions, power series, exponential and logarithmic functions, conformality, Mobius transformations.
Unit II
Complex integration, Cauchy's theorem, Cauchy's integral formula, singularities, Taylor's theorem.
Unit III
The maximum principle, The residue theorem and applications, Riemann mapping theorem.
Unit IV
Invariance of integrals under homotopy, Topology on space of holomorphic and meromorphic functions, Hadamard theory of entire functions, Order of entire functions, Picard's theorem.

Suggested texts :

(1) Donald Sarason, *Complex function theory*, second ed., American Mathematical Society, Providence, RI, 2007.
(2) Gareth A. Jones and David Singerman, *Complex functions*, Cambridge University Press, Cambridge, 1987, An algebraic and geometric viewpoint.
(3) Lars V. Ahlfors, *Complex analysis*, third ed., McGraw-Hill Book Co., New York, 1978, An introduction to the theory of analytic functions of one complex variable, International Series in Pure and Applied Mathematics.
(4) John B. Conway, *Functions of one complex variable*, second ed., Graduate Texts in Mathematics, vol. 11, Springer-Verlag, New York-Berlin, 1978.

4.1.8. *KSM2C04 : Differential Equations.*

Unit I
First and second order equations, general and particular solutions, linear and nonlinear systems, linear independence, solution techniques.

Unit II
Existence and Uniqueness Theorems : Peano's and Picard's theorems, Grownwall's inequality, Dependence on initial conditions and associated flows. Linear system : The fundamental matrix, stability of equilibrium points, Phase-plane analysis, Sturm-Liouvile theory.
Unit III
Nonlinear system and their stability : Lyapunov's method, Non-linear Perturbation of linear systems, Periodic solutions and Poincare-Bendixson theorem.
Unit IV
First order partial differential equation and Hamilton-Jacobi equations. Cauchy problem and classification of second order equations, HolmgrenâĂŹs uniqueness theorem.

Suggested texts :

(1) Philip Hartman, *Ordinary differential equations*, Classics in Applied Mathematics, vol. 38, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2002, Corrected reprint of the second (1982) edition [Birkháuser, Boston, MA; MR0658490 (83e:34002)], With a foreword by Peter Bates.
(2) Lawrence Perko, *Differential equations and dynamical systems*, third ed., Texts in Applied Mathematics, vol. 7, Springer-Verlag, New York, 2001.
(3) Michael Renardy and Robert C. Rogers, A*n introduction to partial differential equations*, second ed., Texts in Applied Mathematics, vol. 13, Springer-Verlag, New York, 2004.
(4) Ordinary Differential Equations", Nandakumaran, A. K.; Datti, P. S.; George, Raju K.

4.2. Elective courses. Elective courses are advanced courses intended to form the foundational material for research in mathematics. They are typically at the level of a PhD coursework. The elective courses carry 4 credits each. Note that all courses listed in the set below need not be offered in every academic year. A list of the elective courses from the following list, offered in an academic year will be announced in the beginning of every academic year and listed in KSoM's website. The elective courses listed are intended for the third and fourth semester.

Elective Courses

Number Theory	Analytic Number Theory
Algebraic Number Theory	Modular Forms
Functional Analysis	Operator Theory
Theory of distributions	Partial Differential Equations
Harmonic Analysis	Advanced Functional Analysis
Measure theoretic probability	Enumerative Combinatorics
Complex Analysis - II	Several Complex Variables
Geometry of Discrete groups	Riemann Surfaces
Algebra - III	
Commutative Algebra	Algebraic Geometry
Representation theory of finite groups	Homological Algebra
Differential Geometry	Riemannian Geometry
Algebraic Topology - I	Algebraic Topology - II
Cryptography	Theory of Computation
Programming	

4.2.1. *KSM3E01 : Number Theory.* The Fundamental Theorem of Arithmetic, Arithmetic Functions and Dirichlet Multiplication, Mean Values of Arithmetic Functions, Characters and Dirichlet Characters, Gauss Sums, Primitive Roots and Quadratic Residues, Primes in Arithmetic Progression, Dirichlet Series and Euler Products, The Riemann Zeta Function $\zeta(s)$, Prime Number Theorem, L-functions, Revisiting the Prime Number Theorem.

Suggested texts :

(1) T.M. Apostol, Introduction to Analytic Number Theory, Undergraduate texts in Mathematics, Springer-Verlag, 1976.
(2) R. Ayoub, An Introduction to the Analytic Theory of Numbers, American Mathematical Society (AMS), 1963.
(3) H. Davenport, Multiplicative Number Theory. Graduate Texts in Mathematics (GTM), Springer-Verlag GTM, 2000.
(4) G.H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers (with an appendix by Andrew Wiles and Roger Heath-Brown), Oxford University Press, 2008.
(5) H. Iwaniec and E. Kowalski, Analytic Number Theory, Colloquium Publications, Vol. 53, American Mathematical Society (AMS), 2004.

(6) G.J.O. Jameson, The Prime Number Theorem. Cambridge University Press, LMS Student texts vol 53, 2003.
(7) Gérald Tenenbaum, Introduction to Analytic and Probabilistic Number Theory: Third Edition, AMS Graduate Studies in Mathematics Volume: 163; 2015.
(8) H.L. Montgomery and R.C. Vaughan, Multiplicative number theory: I. Classical theory. Cambridge University Press, Cambridge Studies in Advanced Math. vol. 97, 2007.
(9) J. Stopple, A Primer of Analytic Number Theory, Cambridge University Press, 2003.

4.2.2. *KSM3E02 : Algebraic Number Theory.* Rings of integers of number fields, Unique factorization of ideals in Dedekind domains, Structure of the group of units of the ring of integers, Finiteness of the class group of the ring of integers, Decomposition and inertia groups, Frobenius elements, Local fields, The product formula, Weak approximation, Ramification, Discriminant and different, Idéles and adéles, Quadratic fields, biquadratic fields, Cyclotomic fields and Fermat's last theorem, How to use a computer to compute many of the above objects (both algorithms and actual use of PARI and MAGMA)

Suggested texts :

(1) Swinnerton-Dyer's A Brief Guide to Algebraic Number Theory
(2) Cassels: Global Fields
(3) Cassels-Frohlich: Algebraic Number Theory
(4) Cohen: A Course in Computational Algebraic Number Theory (GTM 138)
(5) Lang: Algebraic Number Theory (GTM 110)
(6) Fröhlich: Algebraic Number Theory
(7) Marcus: Number Fields
(8) Neukirch: Algebraic Number Theory

4.2.3. *KSM3E03 : Functional Analysis.* Basic topological concepts, Metric spaces, Normed linear spaces, Banach spaces, Bounded linear functionals and dual spaces, Hahn-Banach theorem. Bounded linear operators, open-mapping theorem, closed graph theorem. The Banach-Steinhaus theorem. Hilbert spaces, Riesz representation theorem, orthogonal complements, bounded operators on a Hilbert space up to (and including) the spectral theorem for compact, self-adjoint operators.

Suggested texts :

(1) Rudin, Functional Anaysis (2nd Ed.), McGraw-Hill, 2006.
(2) Yosida, K., Functional Anaysis (4th Edition), Narosa, 1974.
(3) Functional analysis conway
(4) Goffman, C. and Pedrick, G., First Course in Functional Analysis, Prentice-Hall of India, 1995.

4.2.4. *KSM3E04 : Theory of Distributions.* Theory of Distributions: Introduction, Topology of test functions, Convolutions, Schwartz Space, Tempered distributions, Paley-Wiener theorem.

Fourier transform and Sobolev-spaces:Definitions, Extension operators, Continuum and Compact imbeddings, Trace results.

Elliptic boundary value problems: Variational formulation, Weak solutions, Maximum Principle, Regularity results.

Suggested texts :

(1) Barros-Nato, An Introduction to the Theory of Distributions, Marcel Dekker Inc., New York, 1973.
(2) Kesavan S., Topics in Functional Analysis and Applications ,Wiley Eastern Ltd., 1989.
(3) Evans, L. C., Partial Differential Equations, Univ. of California, Berkeley, 1998.
(4) Functional Analysis, Sobolev Spaces and Partial Differential Equations, by Haim Brezis

4.2.5. *KSM3E05 : Harmonic Analysis.* Topological groups, locally compact groups, Haar measure, Modular function, Convolutions, homogeneous spaces, unitary representations, Gelfand-Raikov Theorem. Functions of positive type, GNS construction, Potrjagin duality, Bochner's theorem, Induced representations, Mackey's impritivity theorem.

Suggested texts :

(1) Folland, G. B., A Course in Abstract Harmonic Analysis ,Studies in Advanced Mathematics, CRC Press, 1995.
(2) Hewitt, E and Ross, K., Abstract Harmonic Analysis, Vol. 1 ,Springer 1979.
(3) Gaal, S.A., Linear Analysis and Representation Theory ,Dover, 2010.

4.2.6. *KSM3E06 : Measure Theoretic Probability.* Probability space, Lebesgue measure, Non-measurable sets, Random variables, Borel Probability measures on Euclidean spaces, Examples of probability measures on the line, A metric on the space of probability measures on $\mathbb{R}^d$, Compact subsets of $\mathscr{P}(\mathbb{R}^d)$, Absolute continuity and singularity, Expectation, Limit theorems for Expectation, Lebesgue integral versus Riemann integral Lebesgue spaces, Some inequalities for expectations, Change of variables, Distribution of the sum, product etc. Mean, variance, moments.

Independent random variables, Product measures, Independence Independent sequences of random variables, Some probability estimates, Applications of first and second moment methods, Weak law of large numbers, Applications of weak law of large numbers, Modes of convergence Uniform integrability, Strong law of large numbers, Kolmogorov's zero-one law, The law of iterated logarithm Hoeffding's inequality, Random series with independent terms, Kolmogorov's maximal inequality, Central limit theorem - statement, heuristics and discussion, Central limit theorem - Proof using characteristic functions, CLT for triangular arrays, Limits of sums of random variables Poisson convergence for rare events

Brownian motion, Brownian motion and Winer measure, Some continuity properties of Brownian paths - Negative results, Some continuity properties of Brownian paths - Positive results, Lévy's construction of Brownian motion.

Suggested texts :

(1) Rick Durrett Probability: theory and examples.
(2) Patrick Billingsley Probability and measure, 3rd ed. Wiley India.
(3) Richard Dudley Real analysis and probability, Cambridge university press
(4) Leo Breiman Probability, SIAM: Society for Industrial and Applied Mathematics

4.2.7. *KSM3E07 : Complex Analysis - II.* Automorphisms of Complex and Upper half plane, Analytic continuation, meromorphic continuation along a path, monodromy theorem, Riemann surfaces, branch points, analytic, meromorphic and holomorphic functions on Riemann surfaces.

Conformal Equivalence for Simply Connected Regions, Conformal Equivalence for Finitely Connected Regions, Analytic Covering Maps, De Branges's Proof of the Bieberbach Conjecture, Hardy Spaces on the Disk, Potential Theory in the Plane.

Suggested texts :

(1) Functions of One Complex Variable II, Conway John B.,
(2) Potential Theory - Thomas Ransford
(3) Function theory on planar domains, a second course in complex analysis - D. Fisher
(4) Geometric Function Theory Explorations in Complex Analysis - Krantz

4.2.8. *KSM3E08 : Geometry of Discrete groups.* Möbius Transformations on $\mathbb{R}^n$, Complex Möbius Transformations, Discontinuous Groups, Riemann Surfaces, Hyperbolic Geometry, Fuchsian Groups, Fundamental Domains, Finitely Generated Groups, Universal Constraints On Fuchsian Groups

Suggested texts :

(1) The Geometry of Discrete Groups - Beardon, Alan F.
(2) Fuchsian Groups, Svetlana Katok

4.2.9. *KSM3E09 : Algebra III.* Finite Fields, Number Fields, Galois Theory, Insolvability of degree five equation

Suggested texts :

(1) Number Fields, Markus
(2) Lang, Algebra
(3) Artin, E. Galois Theory
(4) Joseph Rotman, Galois Theory

4.2.10. *KSM3E10 : Commutative Algebra.* Rings and modules, morphisms. Prime spectrum of a ring, tensor products and change of base rings, Localization, noetherian and artinian rings, associated primes and primary decomposition, integral extensions, Noether normalization and Nullstellensatze, dimension theory of noetherian local rings, Krull principal ideal theorem.

Suggested texts :

(1) Introduction to Commutative Algebra - Atiyah M. F. and McDonald I. G.
(2) Commutative Algebra - Matsumura
(3) Commutative Algebra with a View Toward Algebraic Geometry - Eisenbud, David

4.2.11. *KSM3E11 : Representation theory of finite groups.* Generalities on linear representations, Subrepresentations, Irreducible representations, Tensor product of two representations, Symmetric square and alternating square, The character of a representation, Schur's lemma; basic applications, Orthogonality relations for characters, Decomposition of the regular representation, Number of irreducible representations, Canonical decomposition of a representation, Explicit decomposition of a representation, Subgroups, products, induced representations, Abelian subgroups, Product of two groups, Induced representations.

Suggested texts :

(1) Representation Theory, A First Course - Fulton and Harris
(2) Linear Representations of Finite Groups - Jean-Pierre Serre

4.2.12. *KSM3E12 : Differential Geometry.* Differentiable manifolds, differentiable maps, regular values and Sard's theorem, submersions and immersions, tangent and cotangent bundles as examples of vector bundles, vector fields and flows, exponential map, Frobenius theorem, Lie groups and Lie algebras, exponential map, tensors and differential forms, exterior algebra, Lie derivative, Orientable manifolds, integration on manifolds and Stokes' Theorem.

Riemannian metrics, Covariant differentiation, Levi-Civita connection, Curvature and parallel transport, spaces of constant curvature.

Suggested texts :

(1) A comprehensive introduction to differential geometry (Vol. 1) - Spivak M.
(2) Foundations of differentiable manifolds and Lie groups - Frank Warner
(3) Introduction to smooth manifolds - John M Lee

4.2.13. *KSM3E13 : Algebraic Topology - I.* The Fundamental Group, Basic Constructions, Paths and Homotopy, The Fundamental Group of the Circle, Induced Homomorphisms, Van KampenâĂŹs Theorem, Free Products of Groups, The van Kampen Theorem, Applications to Cell Complexes, Covering Spaces, Lifting Properties, The Classification of Covering Spaces, Deck Transformations and Group Actions

Simplicial and Singular Homology, Simplicial Homology, Singular Homology, Homotopy Invariance, Exact Sequences and Excision, The Equivalence of Simplicial and Singular Homology, Computations and Applications, Degree, Cellular Homology, Mayer-Vietoris Sequences, Homology with Coefficients.

Suggested texts :

(1) Algebraic Topology - Alen Hatcher

(2) Algebraic Topology : An Introduction - William S. Massey

4.2.14. *KSM3E14 : Cryptography.* Elementary number theory — Pseudo-random bit generation — elementary cryptosystems — number theoretic algorithms (RSA) — symmetric key cryptosystems - DESIDEA, AES, — authentication — digital signatures, electronic commerce (anonymous cash, micropayments), key management— PGP — zero-knowledge protocols — fairness.

Suggested texts :

(1) N. Koblitz: A course in number theory and cryptography, GTM, Springer.
(2) S. C. Coutinho: The Mathematics of Ciphers, A. K. Peters.
(3) D. Welch: Codes and Cryptography.
(4) W. Stallings: Cryptography and Network Security.

4.2.15. *KSM3E15 : Programming.*

(1) Introduction to basic programming principles using Python
  (a) Names (variables) and values, basic data types.
  (b) Control flow: conditionals, loops, recursion.
  (c) Aggregate data types: lists, dictionaries.
  (d) Input/output, reading files, basic exception handling.
(2) Algorithms and data structures
  (a) Efficiency, big-O notation.
  (b) Sorting and searching.
  (c) Objects and classes.
  (d) Arrays, lists, heaps, binary trees.
(3) Some algorithms/examples drawn from Mathematics
  (a) Matrix operations
  (b) Iterative computations (roots of polynomials etc)
  (c) Geometry in 2D/3D.

Suggested texts :

(1) Mark Pilgrim : Dive into Python, available online
(2) Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein: Introduction to Algorithms, MIT Press (2009).
(3) Jon Kleinberg and Eva Tardos: Algorithm Design, Pearson (2006).
(4) Allen E. Downey: Think Python, available online

4.2.16. *KSM4E01 : Analytic Number Theory.* Sieve methods, Extremal orders, The method of Van der Corput, Generating functions : Dirichlet series, Summation formulae, The Riemann zeta function, The prime number theorem and Riemann hypothesis, The selberg-Delange method, Arithmetic applications, Tauberian Theorems, Prime numbers in arithmetic progressions.

Suggested texts :

(1) Gérald Tenenbaum, Introduction to Analytic and Probabilistic Number Theory: Third Edition, American Mathematical Society (AMS) Graduate Studies in Mathematics Volume: 163; 2015.
(2) H. Iwaniec and E. Kowalski, Analytic Number Theory, Colloquium Publications, Vol. 53, American Mathematical Society (AMS), 2004.
(3) H. Davenport, Multiplicative Number Theory. Graduate Texts in Mathematics (GTM), SpringerâĂŞVerlag GTM, 2000.
(4) G.J.O. Jameson, The Prime Number Theorem. Cambridge University Press, LMS Student texts vol 53, 2003.
(5) John Friedlander and Henryk Iwaniec, Opera De Cribro, Colloquium Publications Volume: 57, American Mathematical Society (AMS), 2010.
(6) H.L. Montgomery and R.C. Vaughan, Multiplicative number theory: I. Classical the- ory. Cambridge University Press, Cambridge Studies in Advanced Math. vol. 97, 2007.
(7) J. Stopple, A Primer of Analytic Number Theory, Cambridge University Press, 2003.

4.2.17. *KSM4E02 : Modular Forms.* The modular group and congruence subgroups, definition of modular forms and first properties, Eisenstein series, theta series, valence formulas, Hecke operators, Atkin-Lehner-Li theory, L-functions, modular curves, modularity. In the final lecture(s) we will give a global outlook on the use of modular forms (and their associated Galois representations) in the solution of Diophantine equations, in particular Fermat's Last Theorem.

Suggested texts :

(1) F. Diamond and J. Shurman, "A First Course in Modular Forms", Graduate Texts in Mathematics 228, Springer-Verlag, 2005. (Covers most of what we will do and much more.)

(2) J.-P. Serre, "A Course in Arithmetic", Graduate Texts in Mathematics 7, Springer-Verlag, 1973. (Chapter VII is very good introductory reading.)
(3) W.A. Stein, "Modular Forms, a Computational Approach", Graduate Studies in Mathematics, American Mathematical Society, 2007. (Emphasis on computations using the free open-source mathematics software system Sage.) See also William A. Stein's Modular Forms Database as well as The L-functions and Modular Forms Database.
(4) J.S. Milne, "Modular Functions and Modular Forms", online course notes.
(5) J.H. Bruinier, G. van der Geer, G. Harder and D. Zagier, "The 1-2-3 of Modular Forms", Universitext, Springer-Verlag, 2008.

4.2.18. *KSM4E03 : Operator Theory.* Review of basic notions from Banach and Hilbert space theory.

Bounded linear operators: Spectral theory of compact, Self adjoint and normal operators, Sturm-Liouville problems, Green's function, Fredholm integral operators.

Unbound linear operators on Hilbert spaces: Symmetric and self adjoint operators, Spectral theory. Banach algebras Gelfand representation theorem. C^*-algebras, Gelfand-Naimark-Segal construction.

Suggested texts :

(1) A course in Functional Analysis - Conway, J. B.
(2) Functional Analysis - Rudin, W.

4.2.19. *KSM4E054 : Partial Differential Equations.* **Introduction:** Order of a PDE, Classification of PDEs into linear, semi-linear, quasilinear, and fully nonlinear equations, Examples of equations from Physics, Geometry, etc. The notion of well-posed PDEs.

First Order PDEs: Method of characteristics, existence and uniqueness results of the Cauchy problem for quasilinear and fully nonlinear equations.

Second Order Linear PDEs in Two Independent Variables: Classification into hyperbolic, parabolic, and elliptic equations, Canonical forms. The method of Separation of Variables for Laplace, Heat, and Wave equations.

Laplace Equation: Definition of Harmonic functions. Mean-value property, Strong maximum principle and Harnack inequality for harmonic functions, Lioville theorem, smoothness of harmonic functions (harmonic functions are real analytic), Fundamental solution, Green's function, Examples for Greens functions like upper half space and ball, Possion's formula. Energy method, Dirichlet Principle, uniqueness using energy method.
Heat Equation: Fundamental solution of heat equation, Duhamel's principle and formula for solution of $u_t - \Delta u = f(x,t), u(x,0) = g(x)$. Weak maximum principle, Heat mean value formula, strong maximum principle, Smoothness of solutions of heat equation, ill-posedness of backward heat equation, Energy methods.
Wave Equation: Well-posedness of initial and boundary value problem in 1D and d'Alembert's formula, Method of descent in 2D and 3D, Duhamel's principle, domain of dependence, range of influence, and finite speed of propagation, energy method.
Real analytic theory: Definition of powerseries in multi dimension, notion of real analytic function and its properties. Cauchy-Kowalevski Theorem, Holmgrem uniqueness theorem.
Limitations of classical solutions and weak solutions: Lewy's example of PDE for which no solution exists Hamilton Jacobi equations, conservation Laws

Suggested texts :

(1) Evans, L. C. *Partial Differential Equations*, AMS, 2010.
(2) Han, Q. *A Basic Course in Partial Differential Equations*, AMS, 2011.
(3) McOwen, R. *Partial Differential Equations: Methods and Applications*, Pearson, 2002.
(4) Pinchover, Y. and Rubinstein, J. *An Introduction to Partial Differential Equations*, Cambridge, 2005.
(5) FritzJohn *Partial Differential Equations*, springer

4.2.20. *KSM4E05 : Advanced Functional Analysis.* Banach algebra of continuous functions, Banach algebra of operators, Abstract Banach algebras, Space of multiplicative linear functionals, Maximal ideal spaces, Gelfand transform, Gelfand-Mazur theorem, Gelfand theorem for commutative Banach algebras, Stone-Weierstrass theorem, Generalised Stone-Weierstrass theorem, Algebra of bounded measurable functions, Gelfand-Naimark theorem, Spectral theorem for C*-algebras, Functional calculus.

Suggested texts :

(1) Banach algebra techniques in operator theory, Ronald G Douglas

(2) An invitation to C*-algebras, W. B. Arveson
(3) Functional analysis, V. S. Sunder
(4) Introduction to topology and modern analysis, G. F. Simmons
(5) C*-algebras and operator theory, G.J. Murphy
(6) Functional Analysis, M. Reed and B. Simon
(7) Linear Operator, General Theory N. Dunford, and T. Schwartz

4.2.21. *KSM4E06 : Enumerative Combinatorics.* Posets, lattices, Mobius inversion, Sieving methods etc; Theory of rational and exponential generating functions; Graph theory.

Suggested texts :

(1) R. Stanley, Enumerative Combinatorics
(2) H. Wilf, Generating functionology.
(3) West, Introduction to Graph Theory

4.2.22. *KSM4E07 : Several Complex Variables.* The $\bar{\partial}$-operator, holomorphic functions, Cauchy's formula, The Hartog's phenomenon the Dolbeault Lemma, Power series and Reinhardt domains, Domains of holomorphy, Subharmonic and plurisubharmonic functions, Convexity and pseudoconvexity, L^2-estimates and consequences, Stein manifolds, Analytic varieties and holomorphic germs, Sheaf cohomology and the Cousin problems, Coherent analytic sheaves

Suggested texts :

(1) Function Theory of Several Complex Variables by S. G. Krantz
(2) An Introduction to Complex Analysis in Several Variables by Lars Hörmander,
(3) Complex Analytic and Algebraic Geometry by Demailly

4.2.23. *KSM4E08 : Riemann Surfaces.* The Definition of Riemann Surfaces, Elementary Properties of Holomorphic Mappings, Homotopy of Curves. The Fundamental Group, Branched and Unbranched Coverings, The Universal Covering and Covering Transformations, Sheaves, Analytic Continuation, Algebraic Functions, Differential Forms, The Integration of Differential Forms, Linear Differential Equations, Compact Riemann Surfaces, Cohomology Groups, Dolbeault's Lemma, A Finiteness Theorem, The Exact Cohomology Sequence, The Riemann-Roch Theorem, The Serre Duality Theorem

Suggested texts :

(1) Lectures on Riemann Surfaces - Otto Forster
(2) Riemann Surfaces - Simon Donaldson
(3) Riemann Surfaces by Way of Complex Analytic Geometry - Dror Varolin

4.2.24. *KSM4E09 : Algebraic Geometry.* Affine Varieties, Projective Varieties, Morphisms, Rational Maps, Nonsingular Varieties, Nonsingular Curves, Intersections in Projective Space.

Suggested texts :

(1) Algebraic Geometry - Hartshorne
(2) Foundations of Algebraic Geometry - Ravi Vakil

4.2.25. *KSM4E10 : Homological Algebra.* Complexes, Homology sequence, Euler characteristic and the Grothendieck group, Injective modules, Homotopies of morphisms of complexes, Derived functors, Delta-functors, Bifunctors, Spectral sequences, Special complexes, Finite free resolutions, Unimodular polynomial vectors, The Koszul complex

Suggested texts :

(1) Homological Algebra - Henri Cartan & Samuel Eilenberg
(2) Introduction to Homological Algebra - Rotman

4.2.26. *KSM4E11 : Riemannian Geometry.* Riemannian metrics, Existence theorems and first examples, Covariant derivative, Geodesics, The curvature tensor, Ricci curvature, scalar curvature, First and second variation, Jacobi vector fields, Riemannian submersions and curvature, Myers' theorem, Cartan-Hadamard's theorem.

Suggested texts :

(1) Riemannian Geometry - Gallot, Sylvestre, Hulin, Dominique, Lafontaine, Jacques
(2) Riemannian Manifolds, An Introduction to Curvature - Lee, John M.

4.2.27. *KSM4E12 : Algebraic Topology - II.* Cohomology Groups, The Universal Coefficient Theorem, Cohomology of Spaces, Cup Product, The Cohomology Ring, A Kunneth Formula, Spaces with Polynomial Cohomology, Poincaré Duality, Orientations and Homology, The Duality Theorem, Connection with Cup Product, Other Forms of Duality, Universal Coefficients for Homology, The General Kunneth Formula.

Suggested texts :

(1) Algebraic Topology - Hathcer

(2) An Introduction to Algebraic Topology - Rotman, Joseph
(3) Algebraic Topology : An Introduction - William S. Massey

4.2.28. *KSM4E13 : Theory of Computation.* Finite automata—regular languages—pumping lemma— stack automata—context free languages—applications to compilers—Turing machines—universal Turing machines— halting problem—non deterministic Turing machines— complexity classes—P v/s NP.

Suggested texts :

(1) J. E. Hopcroft and J. D. Ullman: Introduction to Automata theory, Languages and Computation, Narosa.
(2) D. Kozen: Automata and Computability, Springer.

The syllabus for the elective courses will be updated in KSoM website periodically. Students are advised to pick the elective courses in consultation with her/his thesis advisor.