Kerala School of Mathematics
Course in Statistics for Scientists
Design of Experiments

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6 March 2010

Data Collection Methods

- Unplanned studies (Observational Studies)
  - Data collected from records
  - Data collected without any plan or design
- Census
  - Data collected by a complete coverage of the units involved
- Sample Surveys
  - Data collected according to a specified probability sampling scheme
- Experiments
  - Data collected by a controlled experiment

Surveys and Experiments

- Surveys
  - Surveys are generally meant to estimate population parameters like mean, total, proportion, etc.
  - Randomization is used to select sample units appropriately.
  - No control is exercised on the observed units.
- Experiments
  - Experiments are generally meant to make comparisons.
  - Randomization is used to allocate experimental units or subjects to treatments.
  - Control is exercised on the experimental units.

Ingredients of an Experiment

- Treatments—the entities to be compared (Examples: drugs, fertilizers, processes).
- Experimental Units (Examples: human subjects, rats, plots, machines).
- Response—single or multiple (Examples: decrease in systolic blood pressure, yield of a crop, production).
- Information on the experimental units relevant to the treatment and response (Examples: age of subject, initial weight of rat, water level, make of machine).
- All factors other than those experimented are controlled.
- Experimental units are allotted to treatments.

An Experiment to Compare Drugs for Hypertension

- Three drugs A, B, C are the **treatments**, A, B established drugs, C is a new experimental drug.
- 15 subjects (**experimental units**) are available for the experiment.
- Information on relevant factors (age, life style, dietary habits, etc.) is not available or not used.
- A simple experimental design is to allot 5 subjects to each drug at random (which means all possible allocations of 5 to each is equally likely).
- Reduction is systolic and diastolic blood pressures (the response) are observed after six months of drug use.
- The mean reductions in the three groups are compared using Analysis of Variance.

Completely Randomized Design

- The design mentioned in the study is a **completely randomized design**.
- The $n = 15$ available experimental units are allotted to the $t = 3$ treatments at random.
- In general, it is not necessary that the numbers allotted to the t treatments be equal, although unless other information is available this would be desirable.
- In general, the numbers could be $n_1, n_2, \ldots, n_t$ adding up to n.
- The important point is the **random** allocation.
- This is the **Principle of Randomization**.

Replication

- Recall that $V(\bar{X}) = \frac{\sigma^2}{n}$.
- Thus the larger the sample size n, the more precise are parameter estimates.
- So to get good estimates the sample size should be as large as possible.
- In comparative experiments, this sample size is reflected in the residual degrees of freedom (d.f. of RSS).
- d.f. of RSS reflects the number of observations in excess of the number of parameters estimated.
- The larger this d.f, the more precise are estimates of treatment effects and their comparisons.
- Subject to cost and time considerations, the number of experimental units should be as large as possible.
- This is the **Principle of Replication**.

Experimental Errors

- Besides the sample size, another factor that impacts precision of estimates and comparisons is σ^2.
- σ^2 is dubbed experimental error.
- This contains errors of instrumentation and observational errors.
- Besides it contains variance due to unexplained causes and factors not taken into account in the experiment.
- In our drug comparison problem, the effect studied is drug, but differential effects of factors such as gender, age, life style, etc., add to the overall variance.
- Since these effects are not factored out (unlike the drug effect) they form a part of the RSS and so make the σ^2 larger and make detection of significant treatment (drug) effects difficult.
- So as many factors as possible which impact on the response should be removed by a suitable design and by collecting data on these factors.
- Often these factors themselves are important to study (like life style in our hypertension drug problem) and their interaction with the drug.
- In such a context the experiment should be designed and carried out to study a combination of factors (**a factorial experiment**).

Physical Control

- In our drug experiment, we obtained information on gender and life style during the experiment but not before and did not use it for designing the experiment.
- If left to chance we may not have any or enough subjects in different categories of these factors.
- Ideally, these factors should be deliberately used in designing the experiment.
- Sometimes we need to eliminate the effect of some factors (like initial weight in a growth study) and other times we need to study these effects for themselves or for interaction with the central factor we are studying.
- For instance, if life style interacts with drugs, we may need to recommend different drugs for different life styles.
- If this is not planned, the differential effects will cancel out and drug effect may show false insignificance.
- The idea of explicitly designing for such factors is the **Principle of Physical Control**.

Randomized Block Experiment

- A modification of our completely randomized design to tackle these issues is as follows.
- Instead of an arbitrary set of 18 subjects, get, say 9 with busy life styles and 9 with sedentary life styles.
- Divide the subjects into 6 groups of 3 each in each group, subjects in each group being either B or S in respect of life style and as similar as possible in other factors such as age, gender, etc.
- Such groups which are homogeneous in respect of relevant factors are called **blocks**.
- Within a block, allocate the three drugs at random to the three subjects.
- This is a **Randomized Block Design**.
- There are 3 treatments and 2 blocks, replicated 3 times, making 18 experimental units..
- The analysis of variance is carried out by a two-way analysis of variance.
- Life style differences (with 1 d.f.), treatment differences (with 2 d.f.), and their interaction (with $1 \times 2 = 2$ d.f.) with 12 d.f. for Residual will be the composition of the ANOVA.