Conference on Number Theory

Kerala School of Mathematics, Kozhikode.

Titles and Abstracts of talks

S. D. Adhikari: Monochromatic solutions of linear homogeneous equations and some related questions.

Abstract: We start with a result of Schur which is about monochromatic solutions of the equation $x+y=z$ for any finite colouring of the set of positive integers. After a brief discussion of various generalizations of Schur's theorem including those of Rado and Folkman-Sanders, we here talk about some recent results on some related questions.

U. K. Anandavardhanan: Mod p representations of $GL(2,F)$

Abstract: Mod p representation theory of p-adic reductive groups originated with the work of Barthel and Livne from the mid nineties where they studied irreducible smooth mod p representations of $GL(2,F)$. Though there has been a lot of recent progress, an understanding of the so called supersingular representations - which are the basic building blocks of the theory - is achieved only in very few cases, essentially when $G = GL(2)$ and $F = Q_p$. The case of $GL(2,Q_p)$ follows from the works of Barthel-Livne and Breuil. In this talk we will survey the case of $GL(2,Q_p)$ and indicate an approach to construct some supersingular representations of $GL(2,F)$ where F is an unramified extension of Q_p. This latter work is joint with Arindam Jana.

R. Balasubramanian: TBA

S. Bhattacharya: Finiteness results, q-series expansions and irreducibility of a certain class of modular forms.

Abstract: 'Holomorphic eta quotients' are certain explicit classical modular forms on suitable Hecke subgroups of the full modular group. If the quotient of two holomorphic eta quotients f and g is holomorphic, we call g a factor of f. We show that a holomorphic eta quotient has only finitely many factors. If a holomorphic eta quotient f has no factors other than 1 and itself, we call f irreducible. We shall see that given a Hecke subgroup G of the full modular group, there are only finitely many irreducible holomorphic eta quotients which are modular forms on G. In particular, since 1/2 is the smallest possible weight of a holomorphic eta quotient,

all holomorphic eta quotients of weight 1/2 are irreducible. Zagier conjectured and Mersmann proved that for every holomorphic eta quotient f of weight 1/2, there exists a positive integer m such that f is a rescaling of some eta quotient on the Hecke subgroup of level 12 by m. We shall give an application of this result in precise determination of the n-th coefficient in the q-series expansion of any holomorphic eta quotient of weight 1/2. A recent result on theta series by Lemke-Oliver will follow trivially. Mersmann's theorem also enabled us to check the irreducibility of any holomorphic eta quotient of weight 1. But in general, for higher weights, it is extremely difficult to distinguish between the irreducible holomorphic eta quotients and the reducible ones. However, we shall show that it is not impossible (at least theoretically) by providing an algorithm for checking irreducibility of holomorphic eta quotients.

S. Böcherer: On the arithmetic of vector-valued Siegel modular forms

K. Chakraborty: Real quadratic fields with low class numbers

Abstract: Class number of a number field is one of the fundamental and mysterious objects in algebraic number theory. In this talk, we will discuss some real quadratic fields having small class number. More precisely, we will look at some criteria for class numbers to be 1, 2 and 3 for real quadratic fields of the Richaud-Degert type. This is a joint work with Azizul Hoque and Mohit Mishra.

S. Das: Sup-norm bounds for Siegel cusp forms

Abstract: We would discuss some results on estimates for the sizes of Siegel cusp forms, in particular that of Saito-Kurokawa lifts, on fundamental domains.

S. Ganguly: Rankin-Selberg L-functions and beyond endoscopy

Abstract: We investigate a non-standard approach to proving analytic continuation of the Rankin-Selberg L-function of two primitive forms of different weights that uses the Petersson trace formula and avoids the familiar unfolding technique. Such an approach was first suggested by Langlands in a lecture titled "Endoscopy and beyond". This is a joint work with Ramdin Mawia.

S. Gun: TBA

B. Heim: On the coefficients of powers of the Dedekind eta function

Abstract: Report on recent results towards the vanishing and non-vanishing of powers of the Dedekind eta function dictated by the so called D'Arcais polynomials. The topic involves, the Nekrasov-Hook length formula, the concept of Hurwitz polynomials, and the Lehmer conjecture. Finally we indicate recent results on sign changes.

R. Jayakumar: TBA

K. Kumarasamy: Taylor expansion of Jacobi forms of degree two

J. Meher: Certain identities among eigenforms

A. Mukhopadhyay: Discrepancy of generalized polynomials.

Abstract: In a joint work with O. Ramare and G. K. Viswanadham, we obtain an upper bound for the discrepancy of the sequence $([p(n)\alpha]\beta)_{n\geq 0}$ generated by the generalized polynomial $[p(x)\alpha]\beta$, where $p(x)$ is a monic polynomial with real coefficients, α and β are irrational numbers satisfying certain conditions.

V. Kumar Murty: TBA

M. Ram Murty: The Central Limit Theorem in Number Theory

Abstract: The central limit theorem in probability theory has had tremendous implications in number theory, beginning with the Erdos-Kac theorem generalizing the celebrated theorem of Hardy and Ramanujan regarding the normal number of prime factors of a random natural number. The Erdos-Kac theorem has been generalized into various settings such as to the study of prime divisors of Fourier coefficients of modular forms by M. Ram Murty and V. Kumar Murty in a series of papers written in the 1980's. In recent work, we push this theme further and report on its ubiquity in number theory and algebra. In particular, we will report on recent research done jointly with Arpita Kar and Neha Prabhu.

V. Patankar: Examples of absolutely simple Abelian surfaces whose reductions are isogenous to square of elliptic curves at all places

Abstract: Let D be an indefinite quaternion division algebra over $\mathbf{Q}$ with discriminant Δ. Let A be an absolutely simple Abelian surface defined over a number field K with multiplication by D. Let v be a finite place of K of good reduction for A. Let k_v denote the residue field associated to v. Then, a folklore result says that A_v, the reduction of A modulo v, is isogenous to the square of some elliptic curve $E(v)$ defined over some finite extension of k_v. In this paper, we refine this result to prove that for a large enough (suitably chosen) number field K such an A has good reduction every where, and that for any finite place v of K, with the possible exception of places v dividing Δ, A_v over k_v is isogenous over k_v to the square of some elliptic curve $E(v)$ defined over k_v. We will provide examples of absolutely simple Abelian surfaces having Quaterninonic multiplication defined over number fields whose reductions are isogenous to squares of elliptic curves at ALL finite places.

P. Philippon: TBA

G. Prakash: TBA

D. Prasad: The Herbrand-Ribet theorem and its generalizations.

Abstract: Class number of number fields, in particular that of cyclotomic fields, are of considerable interest to number theorists — and continues to be very enigmatic. We will discuss one of the most basic theorems about class numbers of cyclotomic fields due to Herbrand and Ribet, and what some natural generalizations can be.

R. Raghunathan: Beyond the Selberg class: Classification

Abstract: We will introduce a class of Dirichlet series containing the Selberg class as well as all automorphic L-functions. We will try to generalise the known classification results for the (extended) Selberg class to this setting.

C.S. Rajan: On potentially equivalent Galois representations

Abstract: Suppose ρ_1, ρ_2 are two ℓ-adic Galois representations of the absolute Galois group of a number field, such that the algebraic monodromy group of the first representation is connected, and the representations are locally potentially equivalent at a set of places of positive upper density. We classify such pairs of representations and show that up to twisting by an arbitrary representation, it is given by a pair of representations one of which is trivial and the other abelian.

Consequently, we obtain that the representations are (globally) potentially equivalent, provided one of the following conditions hold: (a) the ranks of the algebraic monodromy groups are equal; (b) the first representation is absolutely irreducible and (c) the algebraic monodromy group of the second representation is also connected. This is joint work with Vijay Patankar.

D. S. Ramana: Quotient and Product Sets of Subsets of the Positive Integers

Abstract: For a set A of non-zero integers, A/A and AA denote the set of rational numbers (and respectively integers) that can be expressed as the ratio (respectively product) of a pair of elements of A. We describe some problems where these ratio and product sets are of interest. We then give a number of results on the cardinalities of A/A and AA for various special subsets A, including random subsets, of the set of the first n positive integers. This talk is based on a pair of papers with J. Cilleruelo and O. Ramaré.

V. P. Ramesh: TBA

P. Rath: TBA

B. Sahu: Rankin-Cohen brackets on Hilbert modular forms and special values of certain Dirichlet series

Abstract: Given a fixed Hilbert modular form, we consider a family of linear maps between the spaces of Hilbert cusp forms by using the Rankin-Cohen brackets and then we compute the adjoint maps of these linear maps with respect to the Petersson scalar product. The Fourier coefficients of the Hilbert cusp forms constructed using this method involve special values of certain Dirichlet series of Rankin-Selberg type associated to Hilbert cusp forms. This is a joint work with Moni Kumari.

A. Sankaranarayanan : Discrete Mean square estimates for Coefficients of Symmetric power L-functions

Abstract: Let f be a primitive holomorphic Hecke eigenform for the full modular group $SL(2, \mathbf{Z})$. Let $L(sym^j f, s)$ be the j-th symmetric power L-function associated to f, and $\lambda_{sym^j f}(n)$ denote its n-th Fourier coefficient. In this paper we prove asymptotic formula for the sums

$$\sum_{n \leq x} \left|\lambda_{sym^3 f}(n)\right|^2 \quad \text{and} \quad \sum_{n \leq x} \left|\lambda_{sym^4 f}(n)\right|^2$$

with improved error terms for $x \geq x_0$ (large). (It is a joint work with Dr. Saurabh Kumar Singh and Prof. K. Srinivas)

J. Sengupta: Non-vanishing of derivatives of L-functions of cusp forms inside the critical strip

Abstract: in this talk we will state and sketch the proof of a theorem on the non-vanishing of derivatives of L functions of cusp forms inside the critical strip.

K. Senthilkumar: Algebraic independence of the values of Weierstrass functions

Abstract: In this talk, we shall show that certain subfields generated by the values of Weierstrass zeta and elliptic functions have transcendence degree at least two over $\mathbf{Q}$.

K. D. Shankhadhar: On certain correspondences between Jacobi forms and elliptic modular forms

Abstract: In this talk, first we recall the Eichler-Zagier and the Shimura-Shintani correspondences between Jacobi cusp forms of scalar index and elliptic cusp forms, and then we discuss

about the generalizations of these correspondences to Jacobi cusp forms of matrix index. This is a report of joint works with B. Ramakrishnan and M. Manickam.

K. Sinha: Spacings between "straightened" Hecke angles

Abstract: In the 1990s, Katz and Sarnak asked if the distribution of the spacings between "straightened" Hecke angles (corresponding to non-CM Hecke newforms) is the same as the distribution of spacings among points in a random sequence picked uniformly and independently in the unit interval. We provide a partial to answer these questions for Hecke angles lying in small subintervals of $[0, 1]$ by focusing on the pair correlation statistics. This is joint work with Baskar Balasubramanyam.

M. K. Tamilselvi: TBA

R. Thangadurai: Distribution of Residues Modulo p

Abstract: I will give a overview of my works related to this topic and present recent works on computing the cardinality of number quadratic residues in some intervals in $(0, p)$.

A. Vatwani: Logarithmic mean values of multiplicative functions

Abstract: A general mean-value theorem for multiplicative functions taking values in the unit disc was given by Wirsing (1967) and Halasz (1968). We consider a certain class of multiplicative arithmetical functions and let $F(s)$ be the associated Dirichlet series. In this setting, we obtain new Halasz-type results for the logarithmic mean value of f. More precisely, we give estimates in terms of the size of $|F(1+1/\log x)|$ and show that these estimates are sharp. As a consequence, we obtain a non-trivial zero-free region for partial sums of L-functions belonging to our class. We also report on some recent work showing that this zero free region is optimal. This is joint work with Arindam Roy.

M. Waldschmidt: On the arithmetic complexity of a polynomial

Abstract: The classical conjecture on algebraic independence of logarithms of algebraic numbers (a special case of Schanuel's conjecture) is equivalent to a conjecture on the rank of matrices with entries logarithms of algebraic numbers. The proof of the equivalence rests on a result of Damien Roy, according to which any polynomial in n variables $X_1, \dots, X_n$ over a field K is the determinant of a square matrix with entries in $K + KX_1 + \cdots + KX_n$ (polynomials of degree ≤ 1). It turns out that this statement of D. Roy is a basic result in Theoretical Computer Science, which yields to the definition of the determinantal complexity of a polynomial. The complexity of computing the permanent of a square matrix has been investigated by Leslie G. Valiant and gives rise to the open problem of comparing VP versus VNP.